38th Summer Symposium in Real Analysis

Prague, July 7–13, 2014

Program of the conference

Book of abstracts

List of participants

Program of the conference

Tuesday – July 8
Morning session

Room C215
Chairman: O. Zindulka
9:00–9:10 Opening
9:10–10:00 **K. Falconer** Projections of Fractals Old and New
10:10–10:30 **A. Máthé** Purely unrectifiable sets are uniformly purely unrectifiable
COFFEE BREAK
11:00–11:20 **L. Moonens** Lebesgue averages on rectangles
11:30–11:50 **E. Liflyand** Amalgam type spaces and integrability of the Fourier transforms
12:00–12:20 **T. Natkaniec** Ideal convergence of sequences of quasi-continuous functions

Afternoon sessions

Room C215
Chairman: J. Spurný
14:30–14:50 **A. Nawrocki** On some classes of generalized almost periodic functions
15:00–15:20 **D. Bugajewska** On lower bounded Λ-variation and its applications
15:30–15:50 **P. Das** On I^K-Cauchy functions
COFFEE BREAK
16:20–16:40 **I. Kupka** Topology can say more
16:50–17:10 **S. Kowalczyk** On continuity in generalized topology
17:20–17:40 **R. Zdunczyk** Simple systems and closure operators
17:50–18:10 **M. Walczyńska** Embeddability properties of metrizable scattered spaces

Room C217
Chairman: A. Nekvinda
14:30–14:50 **G. Horbaczewska** On microscopic sets with respect to sequences of functions
15:00–15:20 **P. Kalemba** On an ideal related to the ideal (v^0)
15:30–15:50 **M. Filipczak** On algebraic properties of supports of probability Bernoulli-like measures
COFFEE BREAK
16:20–16:40 **R. Wiertelak** Comparison of density topologies generated by sequences of intervals tending to zero
16:50–17:10 **W. Wilczynski** Density points, non-canonical form
17:20–17:40 **M. Plotnikov** Q-measures and uniqueness sets for Haar series
17:50–18:10 **J. Plotnikova** Haar series, martingales, and uniqueness theorems

Wednesday – July 9

Morning session

Room C215
Chairman: P. Reardon

9:00–9:50 **J. Schmeling** Multifractal analysis of standard and multiple ergodic averages
10:00–10:20 **A. Nicolau** Oscillation of Hölder continuous functions
COFFEE BREAK
10:50–11:10 **E. D'Aniello** Attractors for iterated function schemes I
11:20–11:40 **T. H. Steele** Attractors for iterated function schemes II
11:50–12:10 **P. Eliaš** Families of sets and functions related to the uniform convergence of characters

Afternoon sessions

Room C215
Chairman: A. Alikhani-Koopaei

14:30–14:50 **P. Sworowski** On path McShane integral
15:00–15:20 **L. Loukotová** About contingent, differentiability and Lipschitz mappings
15:30–15:50 **M. Turowska** About the contingent
COFFEE BREAK
Chairman: T. Natkaniec
16:20–16:40 **J. Wódka** Products of Świątkowski and quasi-continuous functions
16:50–17:10 **G. Kwiecińska** On the Caratheodory superposition of multifunctions and an existence theorem
17:20–17:40 **E. Mainka-Niemczyk** On series expansion of sine and cosine families
17:50–18:10 **V. Olevskii** Localization and completeness in $L_2(\mathbf{R})$

Room C217
Chairman: J. Hejduk

14:30–14:50 **G. A. Monteiro** Functions of bounded semivariation: a survey
15:00–15:20 **M. Bienias** Algebraic structures in some sets of functions
15:30–15:50 **J. Kawabe** Bounded convergence theorem for nonlinear integral functionals
COFFEE BREAK
Chairman: P. Musial
16:20–16:40 **K. Musiał** Convergence in measure of non-measurable functions
16:50–17:10 **G. Oniani** On the convergence of double Fourier-Haar series by dilations of a set
17:20–17:40 **J. Šupina** Ideal version of QN-space can be trivial
17:50–18:10 **F. Strobin** Spaceability of the set of continuous injections from B_{ℓ_p} into ℓ_p with nowhere continuous inverses

THURSDAY – JULY 10
Morning session

Room C215
Chairman: L. Moonens

9:00–9:20 **T. Keleti** Decomposing the real line into Borel sets closed under addition
9:30–9:50 **J. Spurný** Borel sets and functions in topological spaces
10:00–10:20 **P. Allaart** Zero sets and maximum sets of randomized Takagi functions
COFFEE BREAK
10:50–11:10 **J. Hejduk** On topologies generated by sequences of intervals tending to zero
11:20–11:40 **L. Zajíček** Differences of two semiconvex functions
11:50–12:10 **J. P. Fenecios** On a new characterization of Baire-1 functions

FRIDAY – JULY 11
Morning session

Room C215
Chairman: M. Balcerzak

9:00–9:50 **P. Dodos** Approximations of random variables
10:00–10:20 **A. Riviere** Hausdorff dimension and derivatives of nondecreasing functions
COFFEE BREAK
10:50–11:10 **A. Bartoszewicz** Topological and measure properties of some self-similar sets
11:20–11:40 **D. Pokorný** Traces of separately convex functions
11:50–12:10 **P. Reardon** Embeddings of the Ellentuck, dual Ellentuck and Hechler spaces

Afternoon session

Room C215
Chairman: P. Allaart

14:30–14:50 **J. Doleželová** Distributional chaos – recent results
15:00–15:20 **Z. Kočan** On some properties of dynamical systems on one-dimensional spaces
15:30–15:50 **L. Rucká** Waiting times in a queue with m servers
COFFEE BREAK
Chairman: P. Holický
16:20–16:40 **M. Matejdes** Graph and pointwise upper Kuratowski limit of multifunctions
16:50–17:10 **B. Novotný** Cardinal invariants of the Vietoris topology, fine topology and the topology of uniform convergence on $C(X)$
17:20–17:40 **J. Borsík** Strongly quasicontinuous functions
17:50–18:10 **M. Rmoutil** Proximinal subspaces and norm-attaining functionals

SATURDAY – JULY 12

Morning session

Room C215
Chairman: A. Nicolau

9:00–9:50	**E. Matheron** Invariant measures for linear operators
10:00–10:20	**P. Musial** A Stieltjes type extension of the L^r Peron integral
COFFEE BREAK	
10:50–11:10	**F. Tulone** Multiple Kurzweil-Henstock and Perron dyadic integrals
11:20–11:40	**V. Skvortsov** On M-sets and U-sets for system of characters of zero-dimensional compact groups
11:50–12:10	**M. Morales** Some peculiarities about Henstock-Kurzweil integrable functions space and the Fourier Transform

Afternoon session

Room C215
Chairman: T. O'Neil

14:30–14:50	**T. Filipczak** Algebraic differences of binary sequences
15:00–15:20	**R. Filipow** Ideal convergence of sequences of functions
15:30–15:50	**S. Głąb** Large free subgroups of automorphisms groups of ultrahomogeneous spaces
COFFEE BREAK	
Chairman: E. D'Aniello	
16:20–16:40	**Š. Franěk** Relationship between regulated function of two real variables and sequence of regulated functions of one real variable
16:50–17:10	**D. Seco** Complete systems of inner functions in L^∞
17:20–17:40	**D. Fraňková** Regulated functions of multiple variables
17:50–18:10	**M. Balcerzak** Uniform openness of multiplication in Banach spaces L_p

Abstracts

Pandelis Dodos *Approximations of random variables*

We will present some methods of approximation of a given random variable by "simpler" (and consequently, more manageable) functions. We will also discuss applications.

Kenneth Falconer *Projections of Fractals Old and New*

Sixty years ago, in 1954, John Marstrand published a paper entitled 'Some fundamental geometrical properties of plane sets of fractional dimension' which relates the Hausdorff dimension of a set in the plane to the dimensions of its orthogonal projections onto lines. Arguably, this paper marked the start of the area now known as 'Fractal Geometry'. Starting from Marstrand's original theorem, the talk will survey some of the numerous generalisations and specialisations that continue to attract a great deal of interest today.

Etienne Matheron *Invariant measures for linear operators*

A basic question in topological dynamics is to determine whether a given continuous map acting on a reasonable topological space admits interesting invariant probability measures. In this talk, I will address this problem in the specific setting of linear dynamics, i.e. when the transformation is a continuous linear operator acting on a separable Banach space (or, more generally, a Polish topological vector space).

Jörg Schmeling *Multifractal analysis of standard and multiple ergodic averages*

Given a dynamical system and an observable classical multifractal analysis studies the level sets of the time averages of this observable. In many but by far not all situations this is well understood. In "ideal" situations all fractal dimensions coincide and are also attained by the maximal dimension of an invariant measure sitting on these level sets. Multiple ergodic averages arise if one considers the averages of the product of several observables in different time scaling. This question is much less understood. Many new pheomena occur. In particular, the different notions of fractal dimensions give different values, the maximal dimension of invariant measure may be substantially smaller and unexpected phase transitions are present. We will give a survey of the classical theory and introduce some new concepts to study multiple averages. The second part is based on joint work with Fan, Peres, Seuret, Solomyak and Wu.

Pieter Allaart *Zero sets and maximum sets of randomized Takagi functions*

Takagi's continuous but nowhere differentiable function is defined by

$$T(x) = \sum_{n=0}^{\infty} \frac{1}{2^n} \phi(2^n x),$$

where $\phi(x)$ is the distance from x to the nearest integer. In this talk we examine two natural schemes for multiplying the terms in the above series by random signs (while preserving continuity of the limit function). Several results will be presented regarding the set of maximum points and the zero set of the resulting randomized Takagi function. These sets tend to be random fractals, and their almost-sure Hausdorff dimension is of particular interest. This topic offers many opportunities for further research, so the talk will end with a list of open problems.

Marek Balcerzak *Uniform openness of multiplication in Banach spaces L_p*

It is known that the counterpart of the Banach Openness Principle is not valid for bilinear continuous surjections. The respective counterexamples are due to Cohen, Horovitz, Rudin and others. Also, multiplication from $C[0,1] \times C[0,1]$ to $C[0,1]$ is not an open mapping. However, we have proved in [2] that, for any measure space (X, Σ, μ), the multiplication from $L_p \times L_q$ to L_1 (with $p, q \in [0, \infty]$, $1/p + 1/q = 1$) is an open mapping. Moreover, it is a uniformly open mapping [1]. Another result of [1] states that, if (X, Σ, μ) is a topological measure space and X is σ-compact, then the multiplication from $L_1 \times L_\infty^0$ to L_1 is uniformly open where L_∞^0 stands for the space of all members of L_∞ vanishing at ∞. We obtain, as corollaries, the respective results for the sequence spaces ℓ_p.

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Artur Bartoszewicz *Topological and measure properties of some self-similar sets*
Joint work with Taras Banakh, Małgorzata Filipczak, Emilia Szymonik.

Given a finite subset $\Sigma \subset \mathbb{R}$ and a positive real number $q < 1$ we study topological and measure-theoretic properties of the self-similar set $K(\Sigma; q) = \left\{ \sum_{n=0}^{\infty} a_n q^n : (a_n)_{n \in \omega} \in \Sigma^\omega \right\}$, which is the unique compact solution of the equation $K = \Sigma + qK$. The obtained results are applied to studying partial sumsets $E(x) = \left\{ \sum_{n=0}^{\infty} x_n \varepsilon_n : (\varepsilon_n)_{n \in \omega} \in \{0,1\}^\omega \right\}$ of some (multigeometric) sequences $x = (x_n)_{n \in \omega}$.

For a a finite subset $\Sigma \subset \mathbb{R}$ of cardinality $|\Sigma| \geq 2$, we will write it as $\Sigma = \{\sigma_1, \ldots, \sigma_s\}$ for real numbers $\sigma_1 < \cdots < \sigma_s$. Then we denote

$$\operatorname{diam}(\Sigma) = \sigma_s - \sigma_1, \quad \delta(\Sigma) = \min_{i<s}(\sigma_{i+1} - \sigma_i), \quad \text{and} \quad \Delta(\Sigma) = \max_{i<s}(\sigma_{i+1} - \sigma_i).$$

Also put

$$I(\Sigma) = \frac{\Delta(\Sigma)}{\Delta(\Sigma) + \operatorname{diam}\Sigma} \quad \text{and} \quad i(\Sigma) = \inf\{I(B) : B \subset \Sigma,\ \ 2 \le |B|\}.$$

The self-similar sets $K(\Sigma; q)$ where $q \in (0,1)$ have the following properties:

(1) $K(\Sigma; q)$ is an interval if and only if $q \ge I(\Sigma)$;

(2) $K(\Sigma; q)$ is not a finite union of intervals if $q < I(\Sigma)$ and $\Delta(\Sigma) \in \{\sigma_2 - \sigma_1, \sigma_s - \sigma_{s-1}\}$;

(3) $K(\Sigma; q)$ contains an interval if $q \ge i(\Sigma)$;

(4) If $d = \frac{\delta(\Sigma)}{\operatorname{diam}(\Sigma)} < \frac{1}{3+2\sqrt{2}}$ and $\frac{1}{|\Sigma|} < \frac{\sqrt{d}}{1+\sqrt{d}}$, then for almost all $q \in \left(\frac{1}{|\Sigma|}, \frac{\sqrt{d}}{1+\sqrt{d}}\right)$ the set $K(\Sigma; q)$ has positive Lebesgue measure and the set $K(\Sigma; \sqrt{q})$ contains an interval;

(5) $K(\Sigma; q)$ is a Cantor set of zero Lebesgue measure if $q < \frac{1}{|\Sigma|}$ or, more generally, if $q^n < \frac{1}{|\Sigma_n|}$ for some $n \in \mathbb{N}$ where $\Sigma_n = \left\{ \sum_{k=0}^{n-1} a_k q^k : (a_k)_{k=0}^{n-1} \in \Sigma^n \right\}$.

(6) If $\Sigma \supset \{a, a+1, b+1, c+1, b+|\Sigma|, c+|\Sigma|\}$ for some real numbers $a, b, c \in \mathbb{R}$ with $b \ne c$, then there is a strictly decreasing sequence $(q_n)_{n\in\omega}$ with $\lim_{n\to\infty} q_n = \frac{1}{|\Sigma|}$ such that the sets $K(\Sigma; q_n)$ has Lebesgue mesure zero.

Marek Bienias *Algebraic structures in some sets of functions*

We give an introduction into the notion of lineability, algebrability and strong algebrability. We present two general methods: *Independent Bernstein sets method* and *Exponential like function method* that have a huge number of applications in proving algebrability results. It is a part of my PhD Thesis and is a joint work with Artur Bartoszewicz, Szymon Głąb and Małgorzata Filipczak.

Ján Borsík *Strongly quasicontinuous functions*

A function $f\colon \mathbb{R} \to \mathbb{R}$ is quasicontinuous at a point x if for every positive ε and for every neighborhood U of x there is an open nonempty set $G \subset U$ such that $|f(y) - f(x)| < \varepsilon$ for each point $y \in G$. A function f is quasicontinuous if it is such at each point. Quasicontinuous functions need not be measurable. Moreover, the set of points of discontinuity of a quasicontinuous function is of first category however it need not be of measure zero. In the talk, we will investigate classes of functions between continuous and quasicontinuous functions for which the set of discontinuity points is of measure zero or even σ-porous.

Daria Bugajewska *On lower bounded Λ-variation and its applications*

In 1972 Waterman introduced a certain generalization of the bounded variation in the sense of Jordan, namely the so-called Λ-variation. In this talk we are going to discuss the concept of the Λ-variation in the L^1-setting (the so-called lower Λ variation), which allows to deal with the Λ-variation of functions that are equal almost everywhere. We will also present some applications of this type of variation to operator theory and to nonlinear differential and integral equations. In particular, we will show sufficient conditions which guarantee that a convolution operator or a nonautonomous superposition operator maps the space of functions of lower Λ-bounded variation into itself.

Emma D'Aniello and **Timothy H. Steele** *Attractors for iterated function schemes I and II*

Let X be a compact metric space with $\mathcal{S} = \{S_1, \ldots, S_N\}$ a finite set of contraction maps from X to itself. Call a subset F of X an attractor for the iterated function scheme (IFS) $\mathcal{S}$ if $F = \bigcup_{i=1}^{N} S_i(F)$. Working primarily on the unit interval $I = [0, 1]$, we discuss the structure of individual attractors as well as the topological structure of the collection of attractors for IFS.

Pratulananda Das *On $\mathcal{I}^{\mathcal{K}}$-Cauchy functions*

Joint work Martin Sleziak and Vladimir Toma.

In this paper we introduce the notion of $\mathcal{I}^{\mathcal{K}}$-Cauchy function, where $\mathcal{I}$ and $\mathcal{K}$ are ideals on the same set. The $\mathcal{I}^{\mathcal{K}}$-Cauchy functions are a generalization of $\mathcal{I}^*$-Cauchy sequences and $\mathcal{I}^*$-Cauchy nets. We show how this notion can be used to characterize complete uniform spaces and we study how $\mathcal{I}^{\mathcal{K}}$-Cauchy functions and $\mathcal{I}$-Cauchy functions are related.

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Jana Doleželová *Distributional chaos – recent results*

One of the most important extensions of the concept of Li-Yorke chaos is distributional chaos introduced in [1]. This extended definition is much stronger – there are many mappings which are chaotic in sense of Li-Yorke but not distributionally chaotic.
The talk will be devoted to recent results concerning distributional chaos. We show the existence of an invariant distributionally chaotic set [2]. The relation between chaotic pairs and triples will be investigated [3].

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Jonald P. Fenecios *On a new characterization of Baire-1 functions*
Joint work with Emmanuel A. Cabraly and Abraham P. Raccaz.

A function $f\colon \mathbf{R} \to \mathbf{R}$ is said to be Baire-1 if for every open set U the inverse image of U under f is an F_σ set. Equivalently, Henri Lebesgue showed that $f\colon \mathbf{R} \to \mathbf{R}$ is Baire-1 if and only if for each $\epsilon > 0$ there is a sequence of closed sets $\{E_n\}$ such that $\mathbf{R} = \bigcup_{n=1}^{\infty} E_n$ and $\omega_f(E_n) < \epsilon$ for each n where

$$\omega_f(E_n) = \sup|f(x) - f(y)| : \ x, y \in E_n\}.$$

Recently, P. Y. Lee, W. K. Tang and D. Zhao jointly discovered a new characterization of Baire-1 functions involving the usual ϵ-δ formulation. That is, $f\colon \mathbf{R} \to \mathbf{R}$ is Baire-1 if and only if for each $\epsilon > 0$ there is a positive function $\delta\colon \mathbf{R} \to \mathbf{R}^+$ such that for any $x, y \in \mathbf{R}$

$$|x - y| < \min\{\delta(x), \delta(y)\} \Rightarrow |f(x) - f(y)| < \epsilon.$$

In this study, we slightly improve Lebesgue's theorem using the $\epsilon - \delta$ characterization of Baire-1 functions by establishing the following statement: Let D_f be the set of discontinuity of $f\colon \mathbf{R} \to \mathbf{R}$. Then f is Baire-1 if and only if for each $\epsilon > 0$ there is a sequence of closed sets $\{D_n\}$ such that $D_f = \bigcup_{n=1}^{\infty} D_n$ and $\omega_f(D_n) < \epsilon$ for each natural number n. Some simple applications of the new characterization are discussed.

Małgorzata Filipczak *On algebraic properties of supports of probability Bernoulli-like measures*
Joint work with Artur Bartoszewicz and Tomasz Filipczak.

Let $p \in (0, 1/2)$ and μ_p be the distribution of the sum

$$X = \sum_{k=1}^{\infty} \left(\frac{1}{2^k}\right) X_k$$

where X_k, $k \in \mathbb{N}$, is a sequence of independent random variables with $\Pr(X_k = 0) = p$ and $\Pr(X_k = 1) = 1 - p$.

For any $p \in \left(0, \frac{1}{2}\right]$, μ_p is a complete, continuous probability measure, positive on open intervals, and the set

$$A_p = \left\{x = 0, x_1 x_2 x_3 \ldots_{(2)} : \ \lim_{n\to\infty} \frac{x_1 + \ldots + x_n}{n} = p\right\}$$

has a full μ_p measure.

In particular, $\mu_{\frac{1}{2}}$ is equal to the Lebesgue measure λ and $A_{\frac{1}{2}}$ consists of simply normal numbers in base 2.

We show that:

(1) if $p = \frac{1}{2}$ then $A_p + A_p = [0, 1)$,
(2) if $p \in \left(0, \frac{1}{2}\right)$ then $Int(A_p + A_p) = \varnothing$,
(3) if $p \in \left[\frac{1}{4}, \frac{1}{2}\right]$ then $A_p - A_p = [0, 1)$,
(4) if $p \in \left(0, \frac{1}{4}\right)$ then $Int(A_p - A_p) = \varnothing$.

Tomasz Filipczak *Algebraic differences of binary sequences*
Joint work with Małgorzata Filipczak.

Let X_m be the set of all binary sequences with m elements. Sequences from X_m we treat as (binary) numbers from the group $\mathbb{Z}\left(2^m\right) = \{0, \ldots, 2^m - 1\}$. We prove that, if $\frac{1}{4}m < n < \frac{3}{4}m$ then, for any sequence $x \in X_m$ except for $(1, 0, ..., 0) = 2^{m-1}$, there exist sequences $a, b \in X_m$ containing exactly n ones each, and such that $x = b - a$.

This result implies that for any $p \in \left[\frac{1}{4}, \frac{3}{4}\right]$ and any $x \in [0, 1)$ there are a, b such that $x = b - a$ and the density of ones in binary expansions of a and b are equal to p.

Rafał Filipów *Ideal convergence of sequences of functions*

We prove a characterization showing when the ideal pointwise convergence does not imply the ideal equal (aka quasi-normal) convergence. The characterization is expressed in terms of a cardinal coefficient related to the bounding number $\mathfrak{b}$. We also prove a characterization showing when the ideal equal limit is unique. This is joint work with Marcin Staniszewski.

Šimon Franěk *Relationship between regulated function of two real variables and sequence of regulated functions of one real variable*

There is an interesting relationship between regulated functions $f\colon [a, b] \times [0, 1] \to Y$ (where Y is a Banach space) and pointwise convergent sequences $f_n\colon [a, b] \to Y$ defined by $f_n(t) = f(t, 1/n)$.

Dana Fraňková *Regulated functions of multiple variables*

We will define regulated functions $f\colon X \to Y$ (where X, Y are Banach spaces) and we will present their basic properties, including compactness theorem.

Szymon Głąb *Large free subgroups of automorphisms groups of ultrahomogeneous spaces*
Joint work with Filip Strobin.

In this note we consider the following largeness notion of subgroups of S_∞. A group G is large if it contains a free subgroup of $\mathfrak{c}$ generators. We give a necessary condition for countable structure

A to have large group $\mathrm{Aut}(A)$ of authomorphisms of A. It turns out that any countable free subgroup of S_∞ can be extended to large free subgroup of S_∞, and under Martin's Axiom any free subgroup of S_∞ with cardinality less than $\mathfrak{c}$ can be also extended to large free subgroup of S_∞. Finally, if G_n are finitely generated groups, then we obtain that either $\prod_{n\in\mathbf{N}} G_n$ is large or it does not contain free subgroup of uncountably many generators.

Jacek Hejduk *On topologies generated by sequences of intervals tending to zero*
Joint work with Renata Wiertelak.

Let $\mathbf{R}$ be the set of real numbers, $\mathcal{L}$ the family of Lebesgue measurable subsets of $\mathbf{R}$. By $\lambda(A)$ we shall denote the Lebesgue measure of a measurable set A and by $|I|$ the length of an interval I.

Let $J = \{J_n\}_{n\in\mathbf{N}}$ be a sequence of closed intervals tending to zero. It means that $\mathrm{diam}\{\{0\} \cup J_n\} \underset{n\to\infty}{\longrightarrow} 0$.

We shall say that a point $x_0 \in \mathbf{R}$ is a *J–density point* of a set $A \in \mathcal{L}$, if

$$\lim_{n\to\infty} \frac{\lambda(A \cap (J_n + x_0))}{|J_n|} = 1.$$

Let

$$\Phi_J(A) = \{x \in \mathbf{R} : x \text{ is a } J - \text{density point of } A\}$$

and

$$\alpha(J) = \limsup_{n\to\infty} \frac{\mathrm{diam}\{\{0\} \cup J_n\}}{|J_n|}.$$

If $\alpha(J) < \infty$, then Φ_J is a lower density operator and the family

$$\mathcal{T}_J = \{A \in \mathcal{L} : A \subset \Phi_J(A)\}$$

is a topology on $\mathbf{R}$ containing density topology $\mathcal{T}_d$.

If $\alpha(J) = \infty$, then Φ_J is an almost lower density operator and the family

$$\mathcal{T}_J = \{A \in \mathcal{L} : A \subset \Phi_J(A)\}$$

is a topology containing natural topology.

The major idea of presentation is giving properties of topology $\mathcal{T}_J$ for an arbitrary sequence $J = \{J_n\}_{n\in\mathbf{N}}$ of closed intervals tending to zero including the aspect of separation axioms. Some open problems are also included.

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Grażyna Horbaczewska *On microscopic sets with respect to sequences of functions*

DEFINITION 1 ([A1]). A set $E \subset \mathbf{R}$ is microscopic if for each $\epsilon > 0$ there exists a sequence of intervals $\{I_n\}_{n\in\mathbf{N}}$ such that

$$E \subset \bigcup_{n\in\mathbf{N}} I_n \text{ and } \lambda(I_n) \leq \epsilon^n \text{ for } n \in \mathbf{N}.$$

A family of all microscopic sets is a σ-ideal. It is compared with other σ-ideals.

We consider what it causes if we replace a geometric sequence ϵ^n with another one, i.e. we study families of sets defined as follows.

Let $(f_n)_{n\in\mathbf{N}}$ be a sequence of increasing functions $f_n\colon (0,1) \to (0,1)$ such that $\lim_{x\to 0^+} f_n(x) = 0$ and there exists $x_0 \in (0,1)$ such that for every $x \in (0, x_0)$ the series $\sum_{n\in\mathbf{N}} f_n(x)$ is convergent and the sequence $(f_n(x))_{n\in\mathbf{N}}$ is nonincreasing.

DEFINITION 2 ([H]). A set $E \subset \mathbf{R}$ belongs to $\mathcal{M}_{(f_n)}$ if for each $x \in (0,1)$ there exists a sequence of intervals $\{I_n\}_{n\in\mathbf{N}}$ such that

$$E \subset \bigcup_{n\in\mathbf{N}} I_n \text{ and } \lambda(I_n) \leq f_n(x) \text{ for } n \in \mathbf{N}.$$

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Piotr Kalemba *On an ideal related to the ideal* (v^0).

The ideal (v^0) is known in literature and is naturally associated to the structure $[\omega]^\omega$. We consider some counterpart of the ideal (v^0) related to the structure Dense($\mathbb{Q}$) and investigate its combinatorial properties. By the use of the notion of ideal type we prove, that under CH this ideal is isomorphic to (v^0).

Jun Kawabe *Bounded convergence theorem for nonlinear integral functionals*

In this talk, we introduce a new notion of the perturbation of nonlinear integral functionals to formulate a functional form of the convergence theorems for nonlinear integrals in nonadditive measure theory. As its direct consequences, we obtain the bounded convergence theorems for typical nonlinear integrals, which show that the autocontinuity of a nonadditive measure is equivalent to the validity of the bounded convergence theorems for the Choquet, the Sugeno, and the Shilkret integrals as well as their symmetric and asymmetric extensions.

Tamás Keleti *Decomposing the real line into Borel sets closed under addition*

Joint work with Márton Elekes.

We consider decompositions of the real line into pairwise disjoint Borel pieces so that each piece is closed under addition. How many pieces can there be? We prove among others that the number of pieces is either at most 3 or uncountable, and we show that it is undecidable in ZFC and even in the theory $ZFC + \mathfrak{c} = \omega_2$ if the number of pieces can be uncountable but less than the continuum. We also investigate various versions: what happens if we drop the Borelness requirement, if we replace addition by multiplication, if the pieces are subgroups, if we partition $(0, \infty)$, and so on.

Zdeněk Kočan *On some properties of dynamical systems on one-dimensional spaces*

The talk is based on a joint work with Veronika Kurková and Michal Málek. We consider some properties of discrete dynamical systems such as the existence of an horseshoe, the positivity of topological entropy, the existence of a homoclinic trajectory or Lyapunov instability on the set of periodic points. We survey the known relations between the properties in the case of interval, graph and dendrite maps. For example, in all the mentioned cases the existence of an arc horseshoe implies every of considered properties. But we construct a continuous map on a Peano continuum with an arc horseshoe and zero topological entropy.

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Stanisław Kowalczyk *On continuity in generalized topology*

DEFINITION 1. A family Γ of subsets of X is called a generalized topology if $\emptyset \in \Gamma$ and the union of arbitrary subfamily $\mathcal{B} \subset \Gamma$ belongs to Γ.

There are many kinds of types of continuity considered in real analysis connected with the notion of Lebesgue measure which can be described as a continuity in some generalized topology (for example: $\mathcal{UC}_1$ -continuity, preponderant continuity in O'Malley sense, Darboux property). The following technical lemma is needed to obtain this equivalence.

LEMMA 1. Let a family $\{E_s \colon s \in S\}$ of measurable subsets of $\mathbb{R}$ be such that

$$\forall_{s\in S}\forall_{x\in E_s}\overline{d}(E_s,x) > 0.$$

Then the set $E = \bigcup_{s\in S} E_s$ is measurable.

We present some properties of real functions which are continuous in a generalized topology.

THEOREM 1. Let Γ be a generalized topology in X. Then $\{A \in \Gamma \colon \forall_{B\in\Gamma} A \cap B \in \Gamma\}$ is a topology in X.

DEFINITION 2. Let Γ be a generalized topology in X. By $\mathcal{T}_\Gamma$ we denote the topology $\{A \in \Gamma \colon \forall_{B\in\Gamma} A \cap B \in \Gamma\}$ in X and we call it the topology determined by Γ.

THEOREM 2. Let Γ be a generalized topology in X, $x_0 \in X$, $C_\Gamma(x_0)$ be the family of all functions $g \colon X \to \mathbb{R}$ which are Γ-continuous at x_0 and let $f \colon X \to \mathbb{R}$. Then $f + g \in C_\Gamma(x_0)$ for each $g \in C_\Gamma(x_0)$ if and only if f is continuous in $\mathcal{T}_\Gamma$.

THEOREM 3. Let Γ be a generalized topology in X, $x_0 \in X$, $C_\Gamma(x_0)$ be the family of all functions $g \colon X \to \mathbb{R}$ which are Γ-continuous at x_0 and let $f \colon X \to \mathbb{R}$. Then $\min\{f, g\} \in C_\Gamma(x_0)$ for each $g \in C_\Gamma(x_0)$ if and only if f is continuous in $\mathcal{T}_\Gamma$.

THEOREM 4. Let Γ be a generalized topology in X, $x_0 \in X$, $C_\Gamma(x_0)$ be the family of all functions $g \colon X \to \mathbb{R}$ which are Γ-continuous at x_0 and let $f \colon X \to \mathbb{R}$. Then $\max\{f, g\} \in C_\Gamma(x_0)$ for each $g \in C_\Gamma(x_0)$ if and only if f is continuous in $\mathcal{T}_\Gamma$.

Let $C(\Gamma)$ be the set of all functions $f \colon x \to \mathbb{R}$ continuous in a generalized topology Γ.

COROLLARY 1. Let Γ be a generalized topology in X. Then

(1) $\mathbf{A}_{C(\Gamma)} \subset C(\mathcal{T}_\Gamma)$,

(2) $\mathbf{Max}_{C_\Gamma} \subset C(\mathcal{T}_\Gamma)$,
(3) $\mathbf{Min}_{C_\Gamma} \subset C(\mathcal{T}_\Gamma)$.

Under some natural, but rather complicated, conditions on Γ we have equalities in the assertion of the previous corollary.

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Ivan Kupka *Topology can say more*

Some notions in function theory are topological, some notions are algebraic or "metric". We need a metric to be able to work with notions like contractivity, measure of noncompactness, uniform continuity, uniform convergence, lipschitzness. We need more than a topological structure to define periodicity. Do we?

In our talk we will show, that practically all notions mentioned above can be modelled in a topological way. Several author's results on the topic "topologisation of some nontopological notions"will be presented. Most of them have been published, some of them are new.

Two notions of *topological similarity of functions* (multifunctions) can sometimes replace the property "f and g have similar relative derivatives". We will see, that if two multifunctions are topologically similar then they have similar properties:

THEOREM. Let X, Y, Z be Hausdorff topological spaces, be $F\colon X \to Y$, $G : X \to Z$ multifunctions. Let F has a continuous selection $f\colon X \to Y$. If G is F-continuous ("F-similar") on X then it has a continuous selection $g\colon X \to Z$.

Grażyna Kwiecińska *On the Carathéodory superposition of multifunctions and an existence theorem*

Let $I \subset \mathbb{R}$ be an interval and Y a reflexive Banach space. Let $F\colon I \times Y \rightsquigarrow Y$ be a multifunction and $f\colon I \to Y$ a function. We say that F has the (H) property if $F(\cdot, y)$ is a derivative for each $y \in Y$, the family $\{F(x,\cdot)\}_{x\in I}$ is equicontinuous and the family $\{F_f\}_{f\in\mathcal{C}(I,Y)}$ is uniformly integrably bounded, where $F_f(x) = F(x, f(x))$ for $x \in I$, and $\mathcal{C}(I,Y)$ denotes the family of all continuous vector functions $f\colon I \to Y$.

We prove that the Carathéodory superposition F_f is a derivative whenever F has the (H) property and $f \in \mathcal{C}(I,Y)$. Some application of this theorem to the existence of solutions of differential inclusions $f'(x) \in F(x, f(x))$ will be discussed.

Elijah Liflyand *Amalgam type spaces and integrability of the Fourier transforms*

We introduce an amalgam type space, a subspace of $L^1(\mathbb{R}_+)$. Integrability results for the Fourier transform of a function with the derivative from such an amalgam space are proved. As an application, we obtain conditions for the integrability of trigonometric series. Sharpness of Hardy's inequality can be shown within this scope as well.

Lucie Loukotová *Relative absolute continuity*

The aim of this talk is to introduce a generalization of the classical absolute continuity to a relative case, with respect to a subset M of an interval I. This generalization is based on adding more requirements to disjoint systems (a_k, b_k) from the classical definition of absolute continuity – these systems should be not too far from M and should be small relative to some covers of M. We discuss basic properties of relative absolutely continuous functions and compare this class with other classes of generalized absolutely continuous functions (AC, ACG, AC*, ACG*).

Ewelina Mainka-Niemczyk *On series expansion of sine and cosine families*

Let K be a convex cone in a normed linear space X, and let $E_t\colon K \to n(K)$, $F_t\colon K \to n(X)$ for $t \geq 0$. A family $\{E_t : t \geq 0\}$ is called a sine family associated with family $\{F_t : t \geq 0\}$ if

$$E_{t+s}(x) = E_{t-s}(x) + 2F_t(E_t(x)), \qquad 0 \leq s \leq t,\ x \in K,$$

while family $\{F_t : t \geq 0\}$ is called a cosine family if

$$F_0(x) = \{x\}, \quad F_{t+s}(x) + F_{t-s}(x) = 2F_t(F_s(x)), \quad 0 \leq s \leq t,\ x \in K$$

(here, of course, under assumption that values of F_t are in K).

In the talk the necessary and sufficient condition for a family given by some series to be a regular cosine family is presented. Moreover assumptions, under which a regular cosine and sine families can be expressed by series are given.

Milan Matejdes *Graph and pointwise upper Kuratowski limit of multifunctions*

The contribution deals with a connection between an upper Kuratowski limit of a sequence of graphs of multifunctions and an upper Kuratowski limit of a sequence of their values. Namely, we will study under which conditions for a graph (topological) cluster point $[x, y] \in X \times Y$ of a sequence $\{Gr\,F_n : n \in \omega\}$ of graphs of lower quasi continuous multifunctions, y is a vertical (pointwise) cluster point of the sequence $\{F_n(x) : n \in \omega\}$ of values of given multifunctions.

András Máthé *Purely unrectifiable sets are uniformly purely unrectifiable*

Let E be a Borel set in the plane such that for every 1-Lipschitz function $f : \mathbb{R} \to \mathbb{R}$,

$$\mathcal{L}(\{x \in \mathbb{R} : (x, f(x)) \in E\}) = 0$$

where $\mathcal{L}$ denotes Lebesgue measure.

Then for every $\varepsilon > 0$ there exists an open set $G \supset E$ such that for every $(1-\varepsilon)$-Lipschitz function $f : \mathbb{R} \to \mathbb{R}$,

$$\mathcal{L}(\{x \in \mathbb{R} : (x, f(x)) \in G\}) < \varepsilon.$$

This result implies the statement of the title. I will explain the proof which is based on a game and Martin's Borel determinacy theorem.

Giselle Antunes Monteiro *Functions of bounded semivariation: a survey*

Different notions of variation appear when we are dealing with problems in infinite dimension. The semivariation, for instance, is commonly used in the study of convolution, integral equations and measure differential equations. However, we can observe in the literature a lack of material collecting basic results on such a concept. In this work we summarize the present knowledge as well as some remarks and new results on semivariation – such as, its connection with the notion of abstract Kurzweil-Stieltjes integral.

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Laurent Moonens *Lebesgue averages on rectangles*

We shall discuss, in this talk, conditions under which the maximal operator M associated to a sequence of rectangles (Q_i) in the Euclidean plane, defined by the formula:

$$Mf := \sup_i \frac{1_{Q_i}}{|Q_i|} * |f|,$$

satisfies a weak-$(1,1)$ inequality. As we shall see, a bad behaviour of the maximal operator associated to a sequence of "standard" rectangles of the form $Q_i := [0, a_i] \times [0, b_i]$ $(a_i, b_i > 0)$ leads to a bad behaviour of the maximal operator associated to shifted averages on the rectangles $Q_i' := u_i + Q_i$, where the u_i's are arbitrary vectors – this comes from a joint work with J. M. Rosenblatt. If time permits, we shall also discuss some issues when we replace each Q_i by a rectangle Q_i' obtained from Q_i by some rotation of angle θ_i, when (θ_i) is a lacunary sequence of angles.

María Guadalupe Morales Macías *Some peculiarities about Henstock-Kurzweil integrable functions space and the Fourier Transform.*

In this work we study the Fourier Transform using Henstock-Kurzweil intregral. We get if $f \in HK(\mathbb{R}) \cap BV(\mathbb{R})$ (it means, f is integrable-HK and is a bounded variation function) its Fourier Transform (in classical sense) and the integral (HK sense)

$$\int_{\mathbb{R}} f(x) e^{-isx} dx$$

(which it will called Henstock-Fourier Transform) are equal almost everywhere. Moreover we characterize the set $HK(\mathbb{R}) \cap BV(\mathbb{R})$ respect to space $L^2(\mathbb{R})$. Finally we extend Henstock-Fourier Transform over a subspace in $L^p(\mathbb{R})$, with $1 < p < 2$.

Kazimierz Musiał *Convergence in measure of non-measurable functions*
Joint work with M. Balcerzak.

Let (Ω, Σ, μ) be a complete probability space and let X be a Banach space. We introduce the notion of scalar equi-convergence in measure which being applied to sequences of Pettis or Birkhoff integrable functions generates a new convergence theorem.

Tomasz Natkaniec *Ideal convergence of sequences of quasi-continuous functions*
Joint work with Piotr Szuca.

For any Borel ideal $\mathcal{I}$ we describe the $\mathcal{I}$-Baire system generated by the family of quasi-continuous real-valued functions defined on a Baire space. We characterize ideals $\mathcal{I}$ for which ideal and ordinary Baire systems for quasi-continuous functions coincide. Analogous results for the class of continuous functions have been obtained by Laczkovich and Recław in [2] and (independently) Debs and Saint Raymond in [1].

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Adam Nawrocki *On some classes of generalized almost periodic functions*
Joint work with Dariusz Bugajewski.

One of the most important generalizations of almost periodic functions in the sense of Bohr are almost periodic functions in view of the Lebesgue measure, introduced by Stepanov in 1926. In this talk we are going to discuss asymptotic properties of the classical, continuous and unbounded function, defined by the formula

$$f(x) = \frac{1}{2 + \cos x + \cos\left(x\sqrt{2}\right)}, \qquad \text{for } x \in \mathbb{R}.$$

which is almost periodic in view of the Lebesgue measure as well as in the sense of Levitan. Our discussion will be based on diophantine approximation. In particular, we will present the new method of calculating of some limits.

Artur Nicolau *Oscillation of Hölder continuous functions*

Local oscillation of a function satisfying a Hölder condition is considered, and it is proved that its growth is governed by a version of the Law of the Iterated Logarithm.

Branislav Novotný *Cardinal invariants of the Vietoris topology, fine topology and the topology of uniform convergence on $C(X)$*
Joint work with L'ubica Holá.

Let X be a topological space, $C(X)$ be the space of all real valued continuous functions on X. The topology of uniform convergence τ_U on $C(X)$ is a classical one. Despite this fact, its important cardinal invariant, density, seems not to be understood sufficiently. We present its importance from a somewhat broader point of view.

Let τ_Γ be the Vietoris topology on $X \times R$ restricted to $C(X)$, where functions are identified with their graphs. The basis of the topology τ_Γ consists of the sets of the form

$$B(f,\epsilon) = \{g \in C(X); |f(x) - g(x)| < \epsilon(x) \text{ for } x \in X\},$$

where $f \in C(X)$ and ϵ is lower semi continuous real valued function on X. This fact relates the topology τ_Γ to well known fine topology (or m-topology) τ_ω, where ϵ runs through continuous functions, and also to τ_U. We investigate cardinal invariants of τ_Γ, τ_ω and τ_U depending on the properties of X. Since τ_U is metrizable, many cardinal invariants in our interest are equal to its density $d(\tau_U)$. More interestingly also many cardinal invariants on τ_ω like cellularity, density, Lindelöf number and weight, are equal to $d(\tau_U)$, which is why we study this characteristic.

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Victor Olevskii *Localization and completeness in $L_2(\mathbb{R})$*

We give a description of a localization sequence for a determining average sampler.

Giorgi G. Oniani *On the convergence of double Fourier–Haar series by dilations of a set*

There is studied the convergence of partial sums of a double Fourier-Haar series taken by dilations of a given bounded set W on the plane for which the origin is an inner point. From obtained results it follows that for a set W from the quite general family (that for example contains the family of convex sets) there is possible two alternative cases: either Fourier-Haar series of any function $f \in L([0,1]^2)$ is almost everywhere W-convergent or $L \ln^+ L([0,1]^2)$ is the best integral class in which the almost everywhere W-convergence of double Fourier-Haar series is provided. Moreover, there is found the characteristic property for W determining which one among the two cases is realized.

Mikhail G. Plotnikov *Q-measures and uniqueness sets for Haar series*
Julia Plotnikova *Haar series, martingales, and uniqueness theorems*

In 1910 A. Haar introduced the system $\{H_n\}_{n=0}^{\infty}$ of functions on $[0,\,1)$ [1, 3]. The Haar system may be considered also on the dyadic group $\mathbb{G}$.

Let $(S)=\sum_{n=0}^{\infty} a_n H_n(x)$ be any series with respect to the Haar system on the group $\mathbb{G}$, with complex coefficients a_n; S_N $(N=1,2,\ldots)$ be the Nth partial sum of the series (S). It is well-known [2, 4] that the partial sums S_{2^k} form a discrete-time martingale on the filtered probability space $(\Omega,\,\mathcal{F},\,\{\mathcal{F}_k\}_{k=0}^{\infty},\,\mathbf{P})$, where $\Omega:=\mathbb{G}$, $\mathcal{F}$ is the σ-algebra of Borel sets on $\mathbb{G}$, $\mathbf{P}$ is the normed Haar measure on $\mathbb{G}$. Every σ-algebra $\mathcal{F}_k$ contains the empty set and all unions of dyadic intervals of rank k.

An interesting question is, what properties of the Haar system can be extended to martingales on other filtered probability spaces. We study uniqueness problems for pointwise convergence of martingales.

Choose and fix an arbitrary filtered probability space $(\Omega,\,\mathcal{F},\,\{\mathcal{F}_k\}_{k=0}^{\infty},\,\mathbf{P})$.

DEFINITION 1. We say that a set $A\subset\Omega$ is a *set of uniqueness* for martingales (or else *a set of type $\mathcal{U}_M$*), if trivial martingale is only martingale (X_k) which can satisfy $\lim_{k\to\infty} X_k(\omega)=0$ for all $\omega\in\Omega\setminus A$.

Consider the topology τ on Ω, generated by all sets $B\in\mathcal{F}_\infty \overset{\text{def}}{=} \bigcup_{k=0}^{\infty}\mathcal{F}_k$.

THEOREM 1. Suppose that the probability space $(\Omega,\,\mathcal{F},\,\mathbf{P})$ carrying the topology τ is compact. Then every set $U\in\mathcal{F}_\infty$ with $\mathbf{P}(U)=0$ is a set of type $\mathcal{U}_M$. As corollary, $\emptyset$ is a set of type $\mathcal{U}_M$ under the made assumptions.

The theorem 1 can be generalized as follows.

THEOREM 2. Suppose that a probability space $(\Omega,\,\mathcal{F},\,\mathbf{P})$ carrying the topology τ is compact. Assume that a set $U\in\mathcal{F}_\infty$ with $\mathbf{P}(U)=0$, and a random variable ξ with $\mathbf{E}\,|\xi|<\infty$, and a martingale X_k such that $\lim_{k\to\infty} X_k(\omega)=\xi(\omega)$ for all $\omega\in\Omega\setminus U$, are considered. Then

$$X_k=\mathbf{E}\,(\xi\,|\,\mathcal{F}_k)\quad (\mathbf{P}\text{-a.e.})\quad \text{for each } k=0,\,1,\,\ldots,$$

$\mathbf{E}\,(*\,|\,*)$ is a conditional expectation.

Theorems 1 & 2 may be considered as a martingale version of the well-known theorems of Cantor–Young–Bernstein and de la Vallée Poussin [5].

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Dušan Pokorný *Traces of separately convex functions*

In the talk we will discuss the following question: For a function f of two or more variables which is convex in the directions of the coordinate axes, how can its trace $g(x) = f(x, x, \ldots, x)$ look like? In the two-dimensional case, we provide some necessary and sufficient conditions, as well as some examples illustrating that our approach does not seem to be appropriate for finding a characterization in full generality. For a concave function, however, a characterization is established. The results are a joint work with O. Kurka.

Patrick Reardon *Embeddings of the Ellentuck, dual Ellentuck and Hechler spaces*
Joint work with Andrzej Nowik.

The Ellentuck, dual Ellentuck, Hechler and eventually different topologies are associated with, respectively, Mathias forcing, dual Mathias forcing, dominating real forcing and eventually different real forcing. It is known that these spaces satisfy the Baire property.

We have shown that each of the Ellentuck, dual Ellenctuck and Hechler spaces contains a closed homeomorphic copy of the other two. This is not true of the eventually different topology. We have also shown that the Ellentuck topology satisfies the following dichotomy: every perfect set contains a countable perfect set or a closed copy of the classical Sorgenfey line $(0, 1]$. This dichotomy leads to a Marczewski-Burstin representation of the Ellentuck Marczewski-measurable sets in terms of copies of the classical Sorgenfrey line. Our results imply that the dichotomy also holds in the dual Ellentuck and Hechler topologies.

Alain Riviere *Hausdorff dimension and derivatives of nondecreasing functions*

We endow the space Cr of nondecreasing functions on the unit interval I with the uniform metric and consider its subspace Ccr of continuous nondecreasing functions. We consider typical such functions f, in the sense of Baire categories and focus on the Hausdorff dimension of the set of points where the Diny derivative (eventually upper, lower, left, right) of f is respectively infinite, null or positive and finite.

For example, the set of all $t \in I$ at which f has positive and finite Diny upper derivative, is of Hausdorff dimension 1. Our study, and more specifically the result above, is connected with a more geometric question concerning typical convex bodies.

Martin Rmoutil *Proximinal subspaces and norm-attaining functionals*

For a non-reflexive Banach space X and its closed subspace $Y \subset X$ of finite codimension in X, consider the following two sentences:

(1) Y is proximinal in X;
(2) $Y^{\perp} \subset \mathrm{NA}(X) := \{x^* \in X^*;\ \exists x \in B_X : x^*(x) = \|x^*\|\}$.

It is easy to prove (1) $\Longrightarrow$ (2) for any X. We show that for some Banach spaces X the opposite implication holds (e.g. when X is WLUR), and for some it does not (we construct counterexamples). We also provide a negative solution to the long-standing problem of G. Godefroy of 2-lineability of $\mathrm{NA}(X)$.

Lenka Rucká *Waiting times in a queue with m servers*

Assume a queue with m servers and a single waiting line. The next person in line goes to the first available server. In this talk some estimates will be given for the waiting time (time waiting in line) in terms of the customer arrival rate α and the average serving time $\frac{1}{\sigma}$. The main result is that if m_0 is the minimum number of servers required for equilibrium, then for $m_0 + k$ servers, the expected waiting time is less than $\frac{1}{k\sigma}$.

Daniel Seco *Complete systems of inner functions in L^∞*

Recently, Hedenmalm and Montes-Rodríguez showed that the integer powers of $e^{\pi\alpha i x}$ and $e^{\pi\beta i/x}$ span a weak$-*$ dense subspace of L^∞ on the real line (that is, the system is complete) if and only if $\alpha\beta \leq 1$. We study the problem of adding a third function, in the case when $\alpha\beta > 1$, to make the system complete. The functions that we add are of the form $e^{\pi i\gamma/(x-t)}$ for some $t > 0$.

Valentin Skvortsov *On M-sets and U-sets for system of characters of zero-dimensional compact groups*

It is shown that some results on construction of M-sets and U-sets, obtained earlier for Walsh system, can be extended to the general case of the system of characters of zero-dimensional compact Abelian groups. In particular a construction of a perfect M-set whose p-dimensional Hausdorff measure equals zero, with any $p > 0$, is given for this system. One of the important auxiliary result in this construction is the localization theorem of Schneider type. Construction of “thick” U-sets for the above system of characters are also discussed.

Jiří Spurný *Borel sets and functions in topological spaces*

We present a construction of the Borel hierarchy in general topological spaces and its relation to Baire hierarchy. We define Borel classes of mappings, prove the validity of the Lebesgue-Hausdorff-Banach characterization for them and show their relation to Baire classes of mappings on compact spaces. The obtained results are used for studying Baire and Borel order of compact spaces, answering thus one part of a question asked by R. D. Mauldin. We present several examples showing some natural limits of our results in non-compact spaces.

Filip Strobin *Spaceability of the set of continuous injections from B_{ℓ_p} into ℓ_p with nowhere continuous inverses*

Joint work Marek Balcerzak.

Let $p \in (1, \infty)$. I will show that, in the Banach space of all bounded continuous mappings from B_{ℓ_p} (the open unit ball in ℓ_p) into ℓ_p, the subset consisting of all injections with nowhere continuous inverses contains an isometric copy of ℓ_p (in particular, it is spaceable). The proof uses some ideas on Creswells example which gives us a continuous bijection T from ℓ_2 onto a subset of ℓ_2 such that the inverse T^{-1} is discontinuous everywhere.

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Jaroslav Šupina *Ideal version of QN-space can be trivial*

Studying quasi-normal convergence of sequences of real-valued functions, L. Bukovský, I. Recław and M. Repický [1] introduced the notion of a QN-space and showed that from many points of view a QN-space is small. Lately, P. Das and D. Chandra [3] initiated the investigation of ideal version of QN-space, called $\mathcal{J}$QN-space, based on convergence of reals with respect to ideal $\mathcal{J}$ on ω due to H. Cartan [2] and appropriate modification of quasi-normal convergence. In contrary to QN-space, we have shown that any topological space can be a $\mathcal{J}$QN-space for suitable ideal $\mathcal{J}$. Actually, we have shown that for any ideal $\mathcal{J}$ on ω, all pointwise convergent sequences of real-valued functions converge quasi-normally with respect to $\mathcal{J}$ if and only if $\mathcal{J}$ contains an isomorphic copy of the ideal Fin $\times$ Fin on $\omega \times \omega$ defined by

$$\mathrm{Fin} \times \mathrm{Fin} = \{A \subseteq \omega \times \omega;\ |\{n;\ |\{m;\ (n,m) \in A\}| = \aleph_0\}| < \aleph_0\}.$$

In our presentation we focus on this result and its two ideal version. We describe the above class of ideals and we give an example of a tall P-ideal $\mathcal{J}$ and a set of reals which is a $\mathcal{J}$QN-space but which is not a QN-space. Moreover, we discuss the relation between ideal versions of QN-space and an $\mathrm{S}_1(\Gamma, \Gamma)$-space and we discuss the preservation of these properties under well-known ideal orders.

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Francesco Tulone *Multiple Kurzweil-Henstock and Perron dyadic integrals*

Joint work with Valentin Skvortsov.

We consider multiple Walsh and Haar series which are rectangular convergent outside exceptional sets from some class of U-sets, without assuming a priori integrability of the sum in any prescribed sense, and we solve the coefficients problem by finding an appropriate integral to be used in generalized Fourier formulas. The method is based on reducing the coefficients problem to the one of recovering a function from its derivative with respect to the appropriately chosen dyadic derivation basis. The difficulties which should be overcome in applying this method are related to the fact that the primitive we want to recover is differentiable not everywhere but outside some of the above mentioned U-sets which are not countable in a dimension greater then one. We investigate continuity assumptions which should be imposed on the primitive at the points of exceptional sets to guarantee its uniqueness. It turns out that usual continuity with respect to the dyadic basis is not enough for this purpose and we introduce a stronger notion of continuity, which we call *local Saks continuity* with respect to the basis.

The most natural integration process to recover primitives is *Kurzweil-Henstock integral*. We consider continuity properties of the dyadic Kurzweil-Henstock integral in a dimension greater then one and show that it has local Saks continuity. But it solves the problem of recovering a primitive only in the case of countable sets or some other rather “thin” exceptional sets and fails to solve it in the case of the sets we are interested in. So we have to introduce a suitable *Perron-type integral* defined by major and minor functions having local Saks continuity property. We show that multiple Walsh series which converges everywhere outside a U-set of the type we consider here, is the Fourier series of its sum in the sense of this Perron-type integral. The same result, with some additional assumption on the behavior of the coefficients, is obtained for Haar series.

Małgorzata Turowska *About contingent, differentiability and Lipschitz mappings*

DEFINITION 1 ([3]). Let $\emptyset \neq M \subset Z$, where Z is a real normed space, and $z \in \overline{M}$. The set

$$\left\{v \in Z\colon \exists (z_n)_{n\in\mathbb{N}}, z_n \in M, \lim_{n\to\infty} z_n = z, \exists (\lambda_n)_{n\in\mathbb{N}}, \lambda_n > 0\colon \lim_{n\to\infty} \lambda_n(z_n - z) = v\right\}$$

is called a tangent cone to M at z and is denoted $\mathrm{Tan}(M, z)$. Elements of $\mathrm{Tan}(M, z)$ are called vectors tangent to M at z. The set $\mathrm{Tan}(M, z)$ is also called a contingent of M at z ([1], [2]).

Let X, Y be real normed spaces. We give a criterion for the differentiability (in the Fréchet sense) of mapping $f\colon X \to Y$ in terms of a contingent of its graph. We also give a geometric condition for a mapping to be locally Lipschitz.

Let two mappings $f\colon \mathbb{R} \to Y$, $g\colon \mathbb{R} \to Y$ be given. Assume that the contingent of their graphs are known. What can we say about the contingent of the graph of the sum $f + g$? The answer depends on whether Y is finite-dimensional or not.

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Marta Walczyńska *Embeddability properties of metrizable scattered spaces*

In 2005 Gillam in his paper considered dimensional types (in the sense of Fréchet) of countable metrizable spaces. The purpose of our study is to generalize some of his results. If additionally the space is compact then Mazurkiewicz and Siepiński indicated the smallest ordinal number into which one can embed it. But for any countable scattered metrizable space a similar ordinal is not explicitly described. We determined these ordinals.

Renata Wiertelak *Comparison of density topologies generated by sequences of intervals tending to zero*

Let $\mathbf{R}$ be the set of real numbers, $\mathbf{N}$ the set of natural numbers and $\mathcal{L}$ the family of Lebesgue measurable subsets of $\mathbf{R}$. By $\lambda(A)$ we shall denote the Lebesgue measure of a measurable set A and by $|I|$ the length of an interval I.

Let $\langle s\rangle = \{s_n\}_{n\in\mathbf{N}}$ be an unbounded and nondecreasing sequences of natural numbers. We shall say that a point $x_0 \in \mathbf{R}$ is an $\langle s\rangle$–*density point* of a Lebesgue measurable set A if

$$\lim_{n\to\infty} \frac{\lambda\big(A \cap \big[x_0 - \frac{1}{s_n}, x_0 + \frac{1}{s_n}\big]\big)}{\frac{2}{s_n}} = 1.$$

If $A \in \mathcal{L}$, then we denote

$$\Phi_{\langle s\rangle}(A) = \{x \in \mathbf{R} : x \text{ is a } \langle s\rangle\text{-density point of } A\}.$$

Then $\Phi_{\langle s\rangle} : \mathcal{L} \to \mathcal{L}$ is a lower density operator and the family

$$\mathcal{T}_{\langle s\rangle} = \{A \in \mathcal{L} : A \subset \Phi_{\langle s\rangle}(A)\}$$

is a topology on $\mathbf{R}$ containing density topology $\mathcal{T}_d$ (see [3]).

In the paper [4] is presented generalization of notion of $\langle s\rangle$–density point. Let $\mathcal{J} = \{J_n\}_{n\in\mathbf{N}}$ be a sequence of closed intervals tending to zero. It means that $\operatorname{diam}\{\{0\} \cup J_n\} \underset{n\to\infty}{\longrightarrow} 0$.

We shall say that a point $x_0 \in \mathbf{R}$ is a $\mathcal{J}$–*density point* of a set $A \in \mathcal{L}$, if

$$\lim_{n\to\infty} \frac{\lambda(A \cap (J_n + x_0))}{|J_n|} = 1.$$

Let

$$\Phi_{\mathcal{J}}(A) = \{x \in \mathbf{R} : x \text{ is a } \mathcal{J}\text{-density point of } A\}.$$

Then $\Phi_{\mathcal{J}} : \mathcal{L} \to \mathcal{L}$ is an almost lower density operator and the family

$$\mathcal{T}_J = \{A \in \mathcal{L} : A \subset \Phi_{\mathcal{J}}(A)\}$$

is a topology containing natural topology.

The aim of the presentation is to provide the conditions for inclusion between density type topologies.

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Julia Wódka *Products of Świątkowski and quasi-continuous functions*
Joint work with Aleksander Maliszewski.

Let $X \subset \mathbb{R}$ and $f\colon X \to \mathbb{R}$.

- We say that f is *Świątkowski* if for all $a < b$ with $f(a) \neq f(b)$, there is a y between $f(a)$ and $f(b)$ and an continuity point $x \in (a, b)$ such that $f(x) = y$.
- We say that f is *strong Świątkowski* if for all $a < b$ and each y between $f(a)$ and $f(b)$, there is a continuity point $x \in (a, b)$ with $f(x) = y$.
- We say that f is *quasi-continuous*, if for all $a < x < b$ and each $\varepsilon > 0$ there is a nondegenerate interval $I \subset (a, b)$ such that $\operatorname{diam} f[I \cup \{x\}] < \varepsilon$.
- We say that f is *cliquish*, if for all $a < b$ and each $\varepsilon > 0$ there is a nondegenerate interval $I \subset (a, b)$ such that $\operatorname{diam} f[I] < \varepsilon$.

The set $A \subset \mathbb{R}$ is *simply open* if it is the union of an open set and a nowhere dense set.

The purpose of this talk is to present the characterization of products of Świątkowski functions and products of Świątkowski and quasi-continuous functions.

THEOREM. Let $f\colon \mathbb{R} \to \mathbb{R}$. The following are equivalent:

(i) there are an $n \in \mathbb{N}$ and quasi-continuous or Świątkowski functions $g_1, \ldots, g_n$ such that $f = g_1 \ldots g_n$ on $\mathbb{R}$,

(ii) f is cliquish and the set $f^{-1}(0)$ is simply open,

(iii) there are strong Świątkowski function g and Świątkowski function h such that $f = gh$ on $\mathbb{R}$,

(iv) there are quasi-continuous and Świątkowski functions g and h such that $f = gh$ on $\mathbb{R}$,

(v) there are quasi-continuous function g and Świątkowski function h such that $f = gh$ on $\mathbb{R}$,

(vi) there are quasi-continuous functions g and h such that $f = gh$ on $\mathbb{R}$.

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Luděk Zajíček *Differences of two semiconvex functions*

The talk is based on a joint work with V. Kryštof (in preparation). It is proved that real functions on $\mathbb{R}$ which can be represented as the difference of two semiconvex functions with a general modulus (or of two lower C^1-functions, or of two strongly paraconvex functions) coincide with semismooth functions on $\mathbb{R}$ (i.e. those locally Lipschitz functions on $\mathbb{R}$ for which $f'_+(x) = \lim_{t\to x+} f'_+(t)$ and $f'_-(x) = \lim_{t\to x-} f'_-(t)$ for each x). Further, for each modulus ω, we characterise the class DSC_ω of functions on $\mathbb{R}$ which can be written as $f = g - h$, where g and h are semiconvex with modulus $C_1\omega$ for some $C_1 > 0$ as the class of f which are continuous and f'_+ exists and has locally finite $[C_2\omega]$-variation for some $C_2 > 0$. The research was motivated by recent article by J. Duda and L. Zajíček on Gâteaux differentiability of semiconvex functions, in which surfaces described by differences of two semiconvex functions naturally appear.

Rafał Zduńczyk *Simple systems and closure operators*

The connections between (1) generalized derived-set operators, (2) generalized and standard topologies and (3) generalized Thomson's local systems are examined.

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